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Supermembrane with non-Abelian gauging and Chern-Simons quantization

H. Nishino[a], S. Rajpoot[b]

Department of Physics & Astronomy, California State University, 1250 Bellflower Boulevard, Long Beach, CA 90840, USA

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Abstract. We present the non-Abelian gaugings of supermembranes for general isometries for compactifications from eleven-dimensions, starting with an Abelian case as a guide. We introduce a super Killing vector in eleven-dimensional superspace for a non-Abelian group G associated with the compact space B for a general compactification, and couple it to a non-Abelian gauge field on the world-volume. As a technical tool, we use teleparallel superspace with no manifest local Lorentz covariance. Interestingly, the coupling constant is quantized for the non-Abelian group G, due to its non-trivial mapping $\pi_3(G)$.

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1 Introduction

The concept of the simultaneous double-compactification of supermembranes on three-dimensions (3d) with target eleven-dimensions (11D) into superstrings on 2d with target 10D, was first presented in [1]. Since this first observation, it has been well-known that massive Type IIA supergravity in 10D [2] can also arise from the compactification of M-theory in 11D [3], *via* a Killing vector in the direction of the compactifying 11-th coordinate [4]. This mechanism has been elucidated in terms of component language in [4]. Similar mechanisms are expected to work also in many other dimensional reductions [5].

At the present time, however, it is not clear how these component results can be re-formulated in 11D superspace [6, 7] with symmetries for the supermembrane action [8]. For example, the original important significance of supermembranes, such as fermionic κ-invariance [8,9], or the target 11D superspace Bianchi identities (BIds) [6, 7], has not been clarified in component language [4]. Neither is it clear in [4] how a theory as 'unique' as 11D supergravity [3] can accommodate the 'free' mass parameter m, or how it can make itself equivalent to the conventional theory [3], while generating massive Type IIA supergravity in 10D [2] after compactification.

In this paper, we will clarify the significance of the 'free' parameter m in the context of supermembranes [8] on 11D superspace background [7]. We first review the modification of 11D supergravity with the modified fourth-rank field strength by a Killing vector with the free parameter m [4] in component language. We shall see that all the m-terms cancel themselves in the Bianchi identities, when the field strength is expressed in terms of Lorentz indices. We next show how such disappearance of m-effects is reformulated in superspace [6,7] as well. In other words, there is no effect by the m-dependent terms in superspace.

At first glance, this result seems discouraging, because any effect due to the super Killing vector corresponding to the compactification from 11D into 10D turns out to be a 'phantom'. Interestingly, however, we have also found that if we introduce a $U(1)$ gauge field on the supermembrane world-volume with minimal coupling to a super Killing vector ξ^A, this surely results in a physical effect depending on m. We have also found that such couplings necessitate the existence of a Chern-Simons term. We can further generalize this $U(1)$ gauge group for a torus compactification into 10D, to a more general compactification with a more general non-Abelian isometry group. Fortunately, the m-dependent terms do not upset the basic structure of supermembrane action.

Accordingly, for non-Abelian groups G the super Killing vector $\xi^{A\,I}$ carries the adjoint index $I{=}1,2,\ldots,\dim \mathrm{G}$, where G is associated with the compact space B in the compactification $M_{11} \rightarrow M_D \times B$ from 11D into any arbitrary space-time dimension M_D with $D \equiv 11 - \dim B$ [10]. Typical examples are $G = SO(8)$ for $B = S^7$, and $G = SO(6) \times SO(3)$ for $B = S^5 \times S^2$. The simplest choice of $G = SO(2)$ [4] corresponds to the torus compactification with $B = S^1$. The generalized Scherk-Schwarz type [11] dimensional reduction corresponds to $G = SO(11 - D)$ with $B = SL(11 - D, \mathbb{R})/SO(11 - D)$ [5, 12].

As a technical tool, we use a special set of 11D superspace constraints which we call 'teleparallel superspace constraints' [13]. This is because compactifications from 11D most naturally break local Lorentz symmetry, and

[a] e-mail: hnishino@csulb.edu
[b] e-mail: rajpoot@csulb.edu

therefore, teleparallel superspace with no manifest local Lorentz symmetry is more suitable for such a formulation.

Our vector field on the world sheet is neither auxiliary nor composite, but is topological, and different from the auxiliary vector field introduced in the massive type IIA formulation [4]. It is also distinct from the $U(1)$ vector field used in D-brane formulation [14]. We also expect our formulation can be applied to more general extended objects other than supermembranes, such as those in [15].

2 Modified field strengths in component

In this section, we study the effect of the Killing vector ξ^m for the massive branes described in [4] on 11D supergravity in component language. The Killing vector ξ^m is associated with the compactification of 11D supergravity [3] down to 10D massive Type IIA supergravity [2]. We claim that the additional m-dependent terms in a fourth-rank field strength [4] with ξ^m can eventually disappear in its Bianchi identity, when the field strength is expressed with Lorentz indices.

The fourth-rank field strength G_{mnrs} [3] of the potential B_{mnr} is[1] [4],

$$\check{G}_{mnrs} \equiv \tfrac{1}{6}\partial_{[m}B_{nrs]} - \tfrac{1}{8}m\tilde{B}_{[mn}\tilde{B}_{rs]} . \tag{2.1}$$

Here $\tilde{B}_{mn} \equiv \xi^r B_{rmn}$ and $\tilde{\Lambda}_m \equiv \xi^n \Lambda_{nm}$. More generally, any *tilded* field or parameter implies a contraction with ξ^m from the left corresponding to the 'inner contraction' i_ξ in terms of differential forms [4]. The Killing vector ξ^m specifies the 11-th direction of the compactification [4], associated with the Lie-derivatives

$$\mathcal{L}_\xi B_{mnr} \equiv \xi^s\partial_s B_{mnr} + \tfrac{1}{2}\left(\partial_{[m|}\xi^s\right)B_{s|nr]} \overset{*}{=} 0 , \tag{2.2a}$$

$$\mathcal{L}_\xi\, g_{mn} \equiv \xi^r\partial_r g_{mn} + (\partial_{(m|}\xi^r)g_{r|n)} \overset{*}{=} 0 , \tag{2.2b}$$

$$\mathcal{L}_\xi\, e_m{}^a \equiv \xi^n\partial_n e_m{}^a + (\partial_m\xi^n)\, e_n{}^a \overset{*}{=} 0 , \tag{2.2c}$$

$$E_a\xi^b \overset{*}{=} \xi^c C_{ca}{}^b , \tag{2.2d}$$

$$\mathcal{L}_\xi C_{ab}{}^c = \xi^d E_d C_{ab}{}^c \overset{*}{=} 0 , \tag{2.2e}$$

where $E_a \equiv e_a{}^m\partial_m$ and $C_{ab}{}^c$ is the anholonomy coefficient $C_{ab}{}^c \equiv (E_{[a}e_{b]}{}^m)e_m{}^c$ both with *no* Lorentz connection, because we are using the teleparallel formulation. The symbol $\overset{*}{=}$ stands for a relationship associated with the feature of the Killing vector. As we shall show, our engagement of the teleparallel formulation is compatible with the Killing vector condition. Equation (2.2e) can easily be confirmed by (2.2d). As far as the target 11D superspace is concerned, there will be no physical difference between the teleparallel formulation [13] and the conventional formulation [7], as has been explained also in [13].

The real meaning of the m-modification becomes apparent, when we rewrite the field strength $\check{G}_{abcd}$ in terms of local Lorentz indices:

$$\begin{aligned}\check{G}_{abcd} &\equiv +\tfrac{1}{6}E_{[a}B_{bcd]} - \tfrac{1}{4}\check{C}_{[ab|}{}^e B_{e|cd]} + \tfrac{1}{8}m\tilde{B}_{[ab}\tilde{B}_{cd]} \\ &= G_{abcd} - \tfrac{1}{8}m\tilde{B}_{[ab}\tilde{B}_{cd]} ,\end{aligned} \tag{2.3a}$$

$$G_{abcd} \equiv +\tfrac{1}{6}E_{[a}B_{bcd]} - \tfrac{1}{4}C_{[ab|}{}^e B_{e|cd]} , \tag{2.3b}$$

where we have used the modified anholonomy coefficients

$$\check{C}_{ab}{}^c = C_{ab}{}^c + m\tilde{B}_{ab}\xi^c , \tag{2.4}$$

consistent with the torsion $T_{mn}{}^r = -m\tilde{B}_{mn}\xi^r$ in [4]. Here $C_{ab}{}^c$ is the original anholonomy coefficient when $m = 0$ [3]. The 'disappearance' of the m-effect can be understood in terms of the χ-gauge transformation in [4] which we rename as Λ gauge transformation. Explicitly,

$$\delta_\Lambda B_{mnr} = \tfrac{1}{2}\partial_{[m}\Lambda_{nr]} - \tfrac{1}{2}m\tilde{\Lambda}_{[m}\tilde{B}_{nr]} . \tag{2.5}$$

This together with the other related transformations can be expressed a s[2]

$$\begin{aligned}\delta_\Lambda B_{abc} &= +\tfrac{1}{2}E_{[a}\Lambda_{bc]} - \tfrac{1}{2}\check{C}_{[ab|}{}^d\Lambda_{d|c]} + \tfrac{1}{2}m\tilde{\Lambda}_{[a}\tilde{B}_{bc]} \\ &= +\tfrac{1}{2}E_{[a}\Lambda_{bc]} - \tfrac{1}{2}C_{[ab|}{}^d\Lambda_{d|c]} \\ &= \delta_\Lambda B_{abc}\,\Big|_{m=0} ,\end{aligned} \tag{2.6a}$$

$$\begin{aligned}&\delta_\Lambda e_a{}^m = +m\tilde{\Lambda}_a\xi^m , \quad \delta_\Lambda e_m{}^a = -m\tilde{\Lambda}_m\xi^a , \\ &\delta_\Lambda g_{mn} = -m\tilde{\Lambda}_{(m}\xi_{n)} ,\end{aligned} \tag{2.6b}$$

$$\delta_\Lambda \xi^m = 0 , \quad \delta_\Lambda \xi^a = 0 , \tag{2.6c}$$

$$\begin{aligned}&\delta_\Lambda \check{G}_{mnrs} = +\tfrac{1}{6}m\tilde{\Lambda}_{[m}\tilde{G}_{nrs]} , \\ &\tilde{G}_{mnr} \equiv \xi^s\check{G}_{smnr} ,\end{aligned} \tag{2.6d}$$

$$\delta_\Lambda \check{G}_{abcd} = 0 . \tag{2.6e}$$

Most importantly, when written in terms of Lorentz indices, the field strength $\check{G}_{abcd}$ is neutral under the Λ-transformation. On the other hand, $\check{G}_{mnrs}$ is *not* invariant, as (2.6d) shows, in agreement with [4]. The reason is that the elfbein transformation $\delta_\Lambda e_a{}^m$ cancels exactly the contribution of $\delta_\Lambda\check{G}_{mnrs}$. Relevantly, all the m-dependent terms in (2.6a) completely cancel amongst themselves, making the whole expression exactly the same as in the $m = 0$ case.

Relevantly, in component language[3], C, G, $\check{C}$ and $\check{G}$ satisfy the following BIds

$$\tfrac{1}{2}E_{[a}C_{bc]}{}^d - \tfrac{1}{2}C_{[ab|}{}^e C_{e|c]}{}^d \equiv 0 , \tag{2.7a}$$

$$\tfrac{1}{24}E_{[a}G_{bcde]} - \tfrac{1}{12}C_{[ab|}{}^f G_{f|cde]} \equiv 0 , \tag{2.7b}$$

$$\tfrac{1}{2}E_{[a}\check{C}_{bc]}{}^d - \tfrac{1}{2}\check{C}_{[ab|}{}^e\check{C}_{e|c]}{}^d + m\tilde{G}_{abc}\xi^d \equiv 0 , \tag{2.7c}$$

$$\tfrac{1}{24}E_{[a}\check{G}_{bcde]} - \tfrac{1}{12}\check{C}_{[ab|}{}^f\check{G}_{f|cde]} \equiv 0 . \tag{2.7d}$$

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[1] Our notation for the curved (or Lorentz) indices ${}_{m,n,\ldots}$ (or ${}_{a,b,\ldots}$) are the same as in [6]. Also our antisymmetrization is as in [6], e.g., $A_{[m}B_{n]} \equiv A_mB_n - B_nA_m$ with *no* 1/2 in front.

[2] The check-symbol on $\tilde{G}_{abc}$ in (2.6d) is *not* needed, because $\xi^s\check{G}_{smnr} \equiv \xi^s G_{smnr}$.

[3] We note that in the earlier version of this paper, there was a redundant $\tilde{G}\tilde{B}$-term in the $\check{G}$-BId which should not have been there.

Equation (2.7c) and (2.7d) are equivalent to (2.7a) and (2.7b), reflecting again the disappearance of the m-terms in (2.6a). To put it differently, (2.7c) and (2.7d) follow from (2.7a) and (2.7b). In this process, we require that $\tilde{G}_{abc}$ satisfies its 'own' BId[4]

$$\tfrac{1}{6}E_{[a}\tilde{G}_{bcd]} - \tfrac{1}{4}C_{[ab|}{}^{e}\tilde{G}_{e|cd]} \equiv 0\,. \qquad (2.8)$$

Relevantly, we can show that

$$\tilde{G}_{abc} \equiv \xi^{d}G_{dabc} = -\left(\tfrac{1}{2}E_{[a}\tilde{B}_{bc]} - \tfrac{1}{2}\check{C}_{[ab|}{}^{d}\tilde{B}_{d|c]}\right)\,. \qquad (2.9)$$

The first equality is the original definition, while the second one can be confirmed by the use of (2.3b). The overall negative sign is due to our definition of the *tilded* fields.

As has been mentioned before, (2.2d) has no Lorentz connection. The consistency of our teleparallelism is justified by the consistency of the commutator of the E_a's on ξ^c. In fact, we get

$$[E_a, E_b]\xi^c = E_{[a}(E_{b]}\xi^c) = C_{ab}{}^{d}E_d\xi^c + \xi^d E_d C_{ab}{}^{c}\,, \qquad (2.10)$$

where from the middle to the r.h.s., we have used (2.2d) and the BI (2.7a). As desired, the first term on the r.h.s. coincides with the l.h.s., while the last term vanishes, thanks to (2.2e).

We have thus seen that all the m-dependent terms in the $\check{G}_{abcd}$-BId are cancelled, when this field strength is expressed with Lorentz indices. This means that all of these m-dependent terms do not really gener ate any new physical effect within 11D supergravity. In the next section, this aspect will be used as the guiding principle in the superspace reformulation of our component results [6, 7]. This result of no 'physical' effect of the Killing vector [4] in 11D supergravity [3] is not surprising. This is because 11D supergravity [3] is so tight that there is no room for such an additional free parameter m. The necessity of the teleparallel formulation will be elucidated more in the following sections, when the Killing vector is coupled to supermembrane.

3 Modified BIds in superspace

We saw previously that all the m-modified terms in the $\check{G}$-BId were completely absorbed into field redefinitions within 11D. We showed this in terms of the teleparallel formulation. This aspect will now be reformulated in superspace [6, 7] in terms of the so-called teleparallel superspace developed in [13]. Let us start with the unmodified teleparallel superspace with the super anholonomy coefficients C- and the superfield strength G defined by [13][5]

$$C_{AB}{}^{C} \equiv (E_{[A}E_{B)}{}^{M})E_M{}^{C}\,, \qquad (3.1a)$$

$$G_{ABCD} \equiv \tfrac{1}{6}E_{[A}B_{BCD)} - \tfrac{1}{4}C_{[AB|}{}^{E}B_{E|CD)}\,, \qquad (3.1b)$$

satisfying their BIds

$$\tfrac{1}{2}E_{[A}C_{BC)}{}^{D} - \tfrac{1}{2}C_{[AB|}{}^{E}C_{E|C)}{}^{D} \equiv 0\,, \qquad (3.1a)$$

$$\tfrac{1}{24}E_{[A}G_{BCDE)} - \tfrac{1}{12}C_{[AB|}{}^{F}G_{F|CDE)} \equiv 0\,, \qquad (3.1b)$$

where $E_A \equiv E_A{}^{M}\partial_M$ [6]. The superspace constraints with engineering dimensions $d \le 1$ relevant at $m = 0$ are [13]

$$C_{\alpha\beta}{}^{c} = +i(\gamma^c)_{\alpha\beta}\,, \quad G_{\alpha\beta cd} = +\tfrac{1}{2}(\gamma_{cd})_{\alpha\beta}\,, \qquad (3.2a)$$

$$C_{\alpha\beta}{}^{\gamma} = +\tfrac{1}{4}(\gamma_{de})_{(\alpha}{}^{\gamma}C_{\beta)}{}^{de}\,, \quad C_{\alpha}{}^{bc} = -C_{\alpha}{}^{cb}\,, \qquad (3.2b)$$

$$C_{\alpha b}{}^{\gamma} = +\tfrac{i}{144}(\gamma_b{}^{defg}G_{defg} + 8\gamma^{def}G_{bdef})_{\alpha}{}^{\gamma} - \tfrac{1}{8}(\gamma^{cd})_{\alpha}{}^{\gamma}(2C_{bcd} - C_{cdb})\,. \qquad (3.2c)$$

All other independent components at $d \le 1$ such as $G_{\alpha bcd}$ and $C_{\alpha\beta}{}^{\gamma}$ are all zero.

The super Killing vector ξ^M in superspace for the Abelian gauging corresponds to the torus compactification $M_{11} \to M_{10} \times S^1$, specified by the conditions

$$\mathcal{L}_\xi B_{MNP} \equiv \xi^Q\partial_Q B_{MNP} + \tfrac{1}{2}(\partial_{[M|}\xi^Q)B_{Q|NP)} \overset{*}{=} 0\,, \qquad (3.3a)$$

$$\mathcal{L}_\xi\, E_M{}^{A} \equiv \xi^N\partial_N E_M{}^{A} + (\partial_M\xi^N)E_N{}^{A} \overset{*}{=} 0\,, \qquad (3.3b)$$

$$\mathcal{L}_\xi \xi^M \overset{*}{=} 0\,, \qquad (3.3c)$$

$$E_A\xi^B \overset{*}{=} \xi^C C_{CA}{}^{B}\,, \qquad (3.3d)$$

$$\mathcal{L}_\xi C_{AB}{}^{C} = \xi^D E_D C_{AB}{}^{C} \overset{*}{=} 0\,. \qquad (3.3e)$$

These are the teleparallel superspace generalizations of the component case (2.2). Equation (3.3d) is nothing but the rewriting of (3.3b). As in the component case (2.10), we can confirm the consistency of (3.3d) by considering the commutator $[E_A, E_B\}\xi^C$ along with (3.3e), where the details of computations are skipped here.

The BIds for the m-modified system with the Abelian super Killing vector are[6]

$$\tfrac{1}{2}E_{[A}\check{C}_{BC)}{}^{D} - \tfrac{1}{2}\check{C}_{[AB|}{}^{E}\check{C}_{E|C)}{}^{D} + m\tilde{G}_{ABC}\xi^D \equiv 0\,, \qquad (3.4a)$$

$$\tfrac{1}{24}E_{[A}\check{G}_{BCDE)} - \tfrac{1}{12}\check{C}_{[AB|}{}^{F}\check{G}_{F|CDE)} \equiv 0\,, \qquad (3.4b)$$

$$\tfrac{1}{6}E_{[A}\tilde{G}_{BCD)} - \tfrac{1}{4}\check{C}_{[AB|}{}^{D}\tilde{G}_{D|CD)} \equiv 0\,, \qquad (3.4c)$$

where the modified superfield strengths $\check{C}_{AB}{}^{C}$, $\check{G}_{ABCD}$ and $\tilde{G}_{ABC}$ are defined by[7]

$$\check{C}_{AB}{}^{C} \equiv C_{AB}{}^{C} + m\tilde{B}_{AB}\,\xi^C\,, \qquad (3.5a)$$

$$\check{G}_{ABCD} \equiv \tfrac{1}{6}E_{[A}B_{BCD)} - \tfrac{1}{4}\check{C}_{[AB|}{}^{E}B_{E|CD)}$$

[4] The difference between $\check{C}_{ab}{}^{e}$ and $C_{ab}{}^{e}$ does not matter here, due to the identity $\xi^e\tilde{G}_{ecd} \equiv 0$.

[5] As in [6], we use the indices $A, B, \ldots$ for local Lorentz coordinates in superspace, while $M, N, \ldots$ for curved ones.

[6] In an earlier version of this paper, there was a redundant $m\tilde{G}\tilde{B}$-term in the $\check{G}$-BId that should not be there.

[7] The difference between $\check{C}_{AB}{}^{D}$ and $C_{AB}{}^{D}$ does not matter in (3.5c), due to the identity $\xi^D\tilde{B}_{DC} \equiv 0$. The overall negative sign in (3.5c) is caused by our universal definition of the *tilded* superfields, causing a flipping sign.

$$+ \tfrac{1}{8} m \tilde{B}_{[AB} \tilde{B}_{CD)}$$
$$= G_{ABCD} - \tfrac{1}{8} m \tilde{B}_{[AB} \tilde{B}_{CD)} \,, \tag{3.5b}$$
$$\tilde{G}_{ABC} \equiv - \left[\tfrac{1}{2} E_{[A} \tilde{B}_{BC)} - \tfrac{1}{2} \check{C}_{[AB|}{}^{D} \tilde{B}_{D|C)} \right] \,. \tag{3.5c}$$

Any *tilded* superfield symbolizes the i_ξ-operation defined by $\tilde{X}_{A_1 \dots A_n} \equiv \xi^B X_{BA_1 \dots A_n}$. The important point here is that even though the modified BIds (3.4a) and (3.4b) look different from the original ones (3.1), the former is just the rewrite of the latter. In other words, we can 'derive' (3.4a) and (3.4b) from (3.1), under the definition (3.5). In this sense, the m-modified system is equivalent to the original system (3.1), and therefore the same set of constraints (3.2) satisfies (3.4). This also solves the puzzle of the admissibility of the 'free' mass parameter [4] in 11D supergravity. Conventional wisdom is that 11D supergravity is 'unique' in the sense that it excludes such free parameters [16]. For this reason, we can use exactly the same set of constraints (3.2) for our purpose from now on.

We now generalize this Abelian super Killing vector to the non-Abelian case that corresponds to the more general compactification $M_{11} \to M_D \times B$. According to past experiences in the gauging of σ-models [17], we know that the Lie derivative of the Killing vector no longer vanishes, but is proportional to the structure constant. Such a super Killing vector is specified by the conditions

$$\mathcal{L}_{\xi^I} B_{MNP} \equiv \xi^{Q\,I} \partial_Q B_{MNP} + \tfrac{1}{2} \left(\partial_{[M|} \xi^{Q\,I} \right) B_{Q|NP)}$$
$$\stackrel{*}{=} 0 \,, \tag{3.6a}$$
$$\mathcal{L}_{\xi^I} E_M{}^A \equiv \xi^{N\,I} \partial_N E_M{}^A + \left(\partial_M \xi^{N\,I} \right) E_N{}^A$$
$$\stackrel{*}{=} 0 \,, \tag{3.6b}$$
$$\mathcal{L}_{\xi^I} \xi^{M\,J} \equiv \xi^{P\,I} \partial_P \xi^{M\,J} - \xi^{P\,J} \partial_P \xi^{M\,I}$$
$$\stackrel{*}{=} m^{-1} f^{IJK} \xi^{M\,K} \,, \tag{3.6c}$$
$$E_A \xi^{B\,I} \stackrel{*}{=} \xi^{C\,I} C_{CA}{}^B \,, \tag{3.6d}$$
$$\mathcal{L}_{\xi^I} C_{AB}{}^C = \xi^{D\,I} E_D C_{AB}{}^C \stackrel{*}{=} 0 \,. \tag{3.6e}$$

These are the non-Abelian generalizations of (3.3).

In working out the non-Abelian generalization of the modified BIds (3.4). We encounter an obstruction for the $\check{C}$-BId. This is because an m^2-term with the factor $\xi^{E\,I} \tilde{B}_{EC}{}^J \equiv \xi^{E\,I} \xi^{F\,J} B_{FEC} \neq 0$, no longer vanishes in those BIds due to the additional adjoint indices I, J, which were absent in the Abelian case.

Even though we have not yet succeeded in solving this problem, we can still formulate the case of the non -Abelian minimal couplings in supermembrane, due to the uniqueness of the 11D superspace and the unmodified BIds. We do this in the next section. All we need for κ-invariance are relationships like (3.6) with unmodified superfield strengths.

4 Supermembrane with non-Abelian gauging

In the compactification of $M_{11} \to M_{10} \times S^1$ with Abelian gauging, we have seen in 11D superspace that all the new effects due to the m-dependent terms cancel amongst themselves. By the same token, the nontrivial-looking modified BIds turn out to be completely equivalent to conventional ones. This situation is maintained for the more general compactifications $M_{11} \to M_D \times B$. An intuitive explanation is that even though the original 11D are compactified, the original superfield equations are still satisfied, and therefore, the original BIds are not modified after all.

However, the effect of the super Killing vectors corresponding to the compactifications will definitely have non-trivial effects on the supermembrane action in 3d [8]. This is analogous to the gauging effect of any σ-models on $\mathcal{G}/\mathcal{H}$ with minimal couplings for the gauge subgroup $\mathcal{H}$ of $\mathcal{G}$ [17]. In particular, such minimal coupling can be introduced by the world-volume gauge field $A_i{}^I$.

With these preliminaries, the total supermembrane action I on 3d world-volume is

$$I = \int d^3\sigma \Big[+ \tfrac{1}{2} \sqrt{-g} g^{ij} \eta_{ab} \Pi_i{}^a \Pi_j{}^b - \tfrac{1}{2} \sqrt{-g}$$
$$- \tfrac{1}{3} \epsilon^{ijk} \Pi_i{}^C \Pi_j{}^B \Pi_k{}^A B_{ABC} \tag{4.1}$$
$$+ \tfrac{1}{2} m \epsilon^{ijk} \left(F_{ij}{}^I A_k{}^I - \tfrac{1}{3} f^{IJK} A_i{}^I A_j{}^J A_k{}^K \right) \Big] \,.$$

We use the indices $i, j, \ldots = 0, 1, 2$ for the curved coordinates (σ^i) of 3d world-volume, while $(Z^M) = (X^m, \theta^\mu)$ for the 11D superspace coordinates ggrs. The $E_A{}^M$ is the vielbein in 11D superspace, and the pull-back $\Pi_i{}^A$ with the non-Abelian minimal coupling is

$$\Pi_i{}^A \equiv \left(\partial_i Z^M - m A_i{}^I \xi^{M\,I} \right) E_M{}^A$$
$$\equiv \Pi_i^{(0)A} - m A_i{}^I \xi^{A\,I} \,, \tag{4.2}$$

where m is the coupling constant. The original supermembrane action [8] can be recovered in the limit $m \to 0$. The $A_i{}^I = A_i{}^I(\sigma)$ is the non-Abelian gauge field on the world-volume with its field strength

$$F_{ij}{}^I \equiv \partial_i A_j{}^I - \partial_j A_i{}^I + f^{IJK} A_i{}^J A_j{}^K \,, \tag{4.3}$$

with the structure constant f^{IJK} of the gauge group G. Needless to say, the Abelian case is also obtained as a special case by putting the structure constant to zero, with all related adjoint indices deleted.

As for the 11D superspace background, we adopt the teleparallel superspace [13], for the same reason as in the Abelian case. One intuitive reasoning is that it is more natural to use superspace constraints which do not have manifest local Lorentz covariance. One technical reason is that, as we will see, our action loses the fermionic κ-invariance, when there is a Lorentz connection on the background superspace. For the reason mentioned previously, we can use only the un-modified superfield strength G_{ABCD} and $C_{AB}{}^C$ in teleparallel superspace formulation, instead of the m-modified ones.

Interestingly, since the π_3-homotopy mapping of a non-Abelian group is generally non-trivial, the constant m in front of the Chern-Simons term is quantized. Specifically, $\pi_3(G) = \mathbb{Z}$ for $G = SO(n)$ $(n \neq 4)$, $U(n)$ $(n \geq$

2), $SU(n)$ $(n \geq 2)$, $Sp(n)$ $(n \geq 1)$, G_2, F_4, E_6, E_7, or E_8, while $\pi_3(SO(4)) = \mathbb{Z} \oplus \mathbb{Z}$. For Abelian groups, such a mapping is trivial: $\pi_3(SO(2)) = \pi_3(U(1)) = 0$. For the group with $\pi_3(G) = \mathbb{Z}$, the quantization condition is [18]

$$m = \frac{n}{8\pi} \quad (n = \pm 1, \pm 2, \ldots) \,. \tag{4.4}$$

The local non-Abelian invariance of our action is given in terms of the σ-dependent transformation parameter α^I as

$$\delta_\alpha A_i{}^I = +\partial_i \alpha^I + f^{IJK} A_i{}^J \alpha^K \equiv D_i \alpha^I \,, \tag{4.5a}$$

$$\delta_\alpha Z^M = +m\, \alpha^I \, \xi^{MI} \,, \tag{4.5b}$$

$$\begin{aligned} \delta_\alpha \xi^{MI} &= +m\alpha^J \, \xi^{NJ} \partial_N \xi^{MI} \,, \\ \delta_\alpha \xi^A &\overset{*}{=} - f^{IJK} \alpha^J \xi^{A\,K} \,, \end{aligned} \tag{4.5c}$$

$$\begin{aligned} \delta_\alpha E_M{}^A &\overset{*}{=} - m\, \alpha^I \, (\partial_M \xi^{NI}) E_N{}^A \,, \\ \delta_\alpha E_A{}^M &\overset{*}{=} + m\, \alpha^I \, E_A \xi^{MI} \,, \end{aligned} \tag{4.5d}$$

$$\begin{aligned} \delta_\alpha \Pi_i{}^M &= +m\, \alpha^I \, \Pi_i{}^N \partial_N \xi^{MI} \,, \\ \delta_\alpha \Pi_i{}^A &= 0 \,, \end{aligned} \tag{4.5e}$$

$$\delta_\alpha F_{ij} = 0 \,, \tag{4.5f}$$

$$\begin{aligned} \delta_\alpha B_{MNP} &\overset{*}{=} - \tfrac{1}{2} m \alpha^I (\partial_{[M|} \xi^{QI}) B_{Q|NP)} \,, \\ \delta_\alpha B_{ABC} &= 0 \,. \end{aligned} \tag{4.5g}$$

The Abelian case is easily obtained by the truncation of the adjoint indices and the structure constant. All the (super)fields carrying the curved 11D superspace indices transform non-trivially, *except for* $A_i{}^I$. The local invariance $\delta_\alpha I = 0$ under G is easily confirmed, because of the invariances of $\Pi_i{}^A$ and F_{ij}.

Our action is also invariant under the Λ-gauge transformation

$$\begin{aligned} \delta_\Lambda B_{ABC} &= + \tfrac{1}{2} E_{[A} \Lambda_{BC)} - \tfrac{1}{2} C_{[AB|}{}^D \Lambda_{D|C)} \,, \\ \xi^{A\,I} E_A \Lambda_{BC} &\overset{*}{=} 0 \,, \end{aligned} \tag{4.6a}$$

$$\begin{aligned} \delta_\Lambda E_A{}^M &= +m \tilde\Lambda_A{}^I \xi^{M\,I} \,, \\ \delta_\Lambda E_M{}^A &= -m \tilde\Lambda_M{}^I \xi^{A\,I} \,, \end{aligned} \tag{4.6b}$$

$$\begin{aligned} \delta_\Lambda A_i{}^I &= - \Pi_i{}^A \tilde\Lambda_A{}^I \equiv - \tilde\Lambda_i{}^I \,, \\ \tilde\Lambda_A{}^I &\equiv \xi^{B\,I} \Lambda_{BA} \,, \end{aligned} \tag{4.6c}$$

$$\begin{aligned} &\delta_\Lambda \Pi_i{}^A = 0 \,, \quad \delta_\Lambda g_{ij} = 0 \,, \quad \delta_\Lambda Z^M = 0 \,, \\ &\delta_\Lambda \xi^{A\,I} = 0 \,, \quad \delta_\Lambda \xi^{M\,I} = 0 \,. \end{aligned} \tag{4.6d}$$

We have $\delta_\Lambda \Pi_i{}^A = 0$, justifying the minimal coupling in $\Pi_i{}^A$. We easily see that the crucial F_{ij}-linear terms in $\delta_\Lambda I$ are cancelled by the variation of the Chern-Simons term.

We now study the fermionic κ-invariance [8, 9]. Our action I is invariant under

$$\begin{aligned} \delta_\kappa E^\alpha &\equiv (\delta_\kappa Z^M) E_M{}^\alpha = (I + \Gamma)^{\alpha\beta} \kappa_\beta \\ &\equiv [(I + \Gamma)\kappa]^\alpha \,, \end{aligned} \tag{4.7a}$$

$$\begin{aligned} \delta_\kappa E^a &\equiv (\delta_\kappa Z^M) E_M{}^a = 0 \,, \\ \Gamma &\equiv + \tfrac{i}{6\sqrt{-g}} \epsilon^{ijk} \Pi_i{}^a \Pi_j{}^b \Pi_k{}^c \gamma_{abc} \,, \end{aligned} \tag{4.7b}$$

$$\begin{aligned} \delta_\kappa A_i{}^I &= \Pi_i{}^A \xi^{B\,I} (\delta_\kappa E^C) B_{CBA} \equiv \Pi_i{}^A \xi^{B\,I} \Xi_{BA} \\ &\equiv \Pi_i{}^A \tilde\Xi_A{}^I \,, \end{aligned} \tag{4.7c}$$

$$\begin{aligned} \Xi_{AB} &\equiv (\delta_\kappa E^C) B_{CAB} \,, \\ \delta_\kappa E_A{}^M &= (\delta_\kappa E^B) E_B E_A{}^M - m \tilde\Xi_A{}^I \xi^{M\,I} \,, \\ \delta_\kappa E_M{}^A &= (\delta_\kappa E^B) E_B E_M{}^A + m \tilde\Xi_M{}^I \xi^{A\,I} \,, \end{aligned} \tag{4.7d}$$

$$\delta_\kappa \xi^{A\,I} = (\delta_\kappa E^C) \xi^{B\,I} C_{BC}{}^A \,, \tag{4.7e}$$

$$\delta_\kappa \Pi_i{}^A = \partial_i (\delta_\kappa E^A) + (\delta_\kappa E^C) \Pi_i{}^B C_{BC}{}^A \,, \tag{4.7f}$$

$$\delta_\kappa B_{ABC} = (\delta_\kappa E^D) E_D B_{ABC} \,. \tag{4.7g}$$

As stated in [13], (4.7f) takes a simpler form than in the Lorentz covariant formulation [7]. Needless to say, $\Pi_i{}^A$ in this equation contains the m-term, but still no m-explicit term arises in (4.7f). As can readily be checked, the m-dependent terms in (4.7d) and $\delta_\kappa A_i$ are the special cases of the Λ-transformation rules (4.6b) and (4.6c) with $\Lambda_{AB} \equiv -\Xi_{AB} \equiv -(\delta_k E^C) B_{CAB}$. Note, however, that $\delta_\kappa B_{ABC}$ has *no* such a corresponding term. The effect of having the Xi-terms only for $\delta_\kappa E_M{}^A$, $\delta_\kappa E_A{}^M$ and $\delta_\kappa A_i$ is to cancel the unwanted terms arising in $\delta_\kappa I$ otherwise.

The κ-invariance of our action can be confirmed in a way parallel to the original supermembrane case [8], with subtle differences arising due to the m-dependence and the non-Abelian feature of super Killing vectors. The algebraic g_{ij}-field equation takes exactly the same form as the embedding condition in the conventional case [8]:

$$g_{ij} \doteq \Pi_i{}^a \Pi_{ja} \,, \tag{4.8}$$

where $\doteq$ is for a field equation. Needless to say, our pullbacks contain also the m-dependent terms. Other relationships involving Γ are exactly the same as the conventional case [8] or the Abelian case:

$$\begin{aligned} &\Gamma^2 \doteq +I \,, && \epsilon^{ijk} \gamma_{jk} \Gamma \doteq -2i\sqrt{-g}\, \gamma_i \,, \\ &\gamma_i \equiv + \Pi_i{}^a \gamma_a \,, && \gamma_{ij} \equiv \Pi_i{}^a \Pi_j{}^b \gamma_{ab} \,. \end{aligned} \tag{4.9}$$

As in the Abelian gauging, the confirmation $\delta_\kappa I = 0$ also needs important relationships, such as

$$\partial_{[i} \Pi_{j]}{}^A = \Pi_i{}^B \Pi_j{}^C C_{CB}{}^A - m F_{ij}{}^I \xi^{A\,I} \,, \tag{4.10a}$$

$$\mathcal{L}_\xi B_{ABC} = \xi^{D\,I} E_D B_{ABC} \overset{*}{=} 0 \,. \tag{4.10b}$$

The latter is confirmed by (3.6a), while (4.10a) needs the relationship

$$m \xi^{B\,I} \xi^{C\,J} C_{CB}{}^A \overset{*}{=} f^{IJK} \xi^{A\,K} \,, \tag{4.11}$$

derived from (3.6c). The Abelian gauging can be obtained by truncating the adjoint indices and the structure constants.

One of the most crucial cancellation in the invariance $\delta_\kappa I = 0$ arises out of the Wess-Zumino-Witten term:
(i) From the partial integration of ∂_i in

$$\epsilon^{ijk} \left[\partial_i \left(\delta_\kappa E^C\right)\right] \Pi_j{}^B \Pi_k{}^A B_{ABC}$$

hitting $\Pi_j{}^B$ producing a term with mF_{ij}.
(ii) From the variation $\delta_\kappa A_i$ in the Chern-Simons term, yielding a term with $m\epsilon^{ijk}\tilde{\Xi}_i F_{jk}$. Both of these have the same structure and therefore cancel each other. This cancellation also justifies the necessity of a constant m in the Chern-Simons term, which also serves as the minimal coupling constant at the same time.

As we have seen, it is not only the Λ-invariance, but also the κ-invariance that necessitates the Chern-Simons term. There are other reasons that require the Chern-Simons term. For example, if there were no Chern-Simons term, the minimal couplings of A_i to the superspace coordinates Z^M or g_{ij} would result in additional constraints, spoiling the original physical degrees of freedom of these fields. Thanks to our Chern-Simons term, such constraints will not arise, but all the minimal coupling terms contribute only as the source term J^i to the vector field equation as $\epsilon^{ijk} F_{jk}{}^I \doteq J^{i\,I}$. This also makes the whole system nontrivial, because our newly-introduced gauge field couples to the conventional fields Z^M in a nontrivial way, still respecting the original degrees of freedom.

We have been using teleparallel superspace as the consistent background for our supermembrane modified by the super Killing vector ξ^A. The most important technical reason is the problem with conventional constraints arising from the κ-invariance of our action that should be addressed here. Suppose we adopt Lorentz covariant formulation, replacing (3.3d) and (4.10a) now by

$$\nabla_A \xi^{B\,I} \overset{*}{=} \xi^{C\,I} T_{CA}{}^B \,, \tag{4.12a}$$

$$\nabla_{[i}\Pi_{j]}{}^A = \Pi_i{}^B \Pi_j{}^C T_{CB}{}^A - mF_{ij}{}^I \xi^{A\,I} + mA_{[i}{}^I \Pi_{j]}{}^C \xi^{B\,I} \omega_{BC}{}^A \,, \tag{4.12b}$$

where ∇_i is a Lorentz covariant derivative acting like $\nabla_i X_A \equiv \partial_i X_A + \Pi_i{}^A \omega_{AB}{}^C X_C$. Note that the last term in (4.12b) arises from the difference between $\Pi_{[i|}^{(0)B} \omega_B{}^{AC} \Pi_{|j]C}$ and $\Pi_{[i|}{}^B \omega_B{}^{AC} \Pi_{|j]C}$. Now the problem is that when we vary our action under δ_κ, the Wess-Zumino-Witten term yields an additional term proportional to

$$m\epsilon^{ijk} \Pi_i{}^C A_j \Pi_k{}^D \xi^F \omega_{FD}{}^B \left(\delta_\kappa E^E\right) B_{EBC}$$

that has no other counter-terms to cancel. On the other hand, teleparallel superspace has *no* such an ω-dependent term generated, thanks to the absence of manifest local Lorentz covariance from the outset.

As far as the target 11D superspace is concerned, there is no physical difference between teleparallel superspace [13] and the conventional superspace [7]. However, when it comes to the physics of supermembranes on 3d, we see such a great difference due to the valid fermionic κ-invariance of the action. This seems to tell us that only teleparallel superspace [13] with no manifest local Lorentz covariance is the most suitable and consistent framework with the super Killing vector introduced for the compactification from 11D into 10D. Since the supermembrane is an important 'probe' of superspace background, our result indicates the importance of teleparallel superspace for the compactifications of 11D or M-theory itself.

Before concluding this section, we list here all the field equations of our fields g_{ij}, Z^M and $A_i{}^I$ in 3d:

$$g_{ij} \doteq \Pi_i{}^a \Pi_{ia} \,, \tag{4.8}$$

$$\delta_A{}^a \partial_i \left(\sqrt{-g}\, \Pi^i{}_a\right) - \sqrt{-g}\, \Pi_i{}^B C_{BA}{}^d \Pi^i{}_d \doteq + \tfrac{1}{3} \epsilon^{Ijk} \Pi_i{}^D \Pi_j{}^C \Pi_k{}^B G_{BCDA} - m\epsilon^{ijk} F_{ij}{}^I \xi^{B\,I} \Pi_k{}^C B_{CBA} \,, \tag{4.13a}$$

$$\epsilon^{ijk} \left(F_{jk}{}^I - \tilde{B}_{jk}\right) \doteq \sqrt{-g}\, \Pi^{ia} \xi_a{}^I \,. \tag{4.13b}$$

Compared with the original supermembrane case [8], the A-field equation is extra, while the super Killing vector containing terms represent the new effects. All other terms are formally the same as in the $m = 0$ case.

The mutual consistency between (4.13a) and (4.13b) can be confirmed by taking the divergence of the latter. In fact, we get

$$\begin{aligned} 0 &\overset{?}{\doteq} D_i \left(\epsilon^{ijk} F_{jk}{}^I - \sqrt{-g}\Pi^{ia} \xi_a{}^I + \epsilon^{ijk} \xi^{A\,I} \Pi_j{}^C \Pi_k{}^B B_{BCA}\right) \\ &= - \left[\partial_i \left(\sqrt{-g}\, \Pi^{ia}\right)\right] \xi_a{}^I - \sqrt{-g}\, \Pi^{ia} \Pi_i{}^B \xi^{E\,I} C_{EB}{}^a \\ &\quad - m\epsilon^{ijk} \xi^{A\,I} F_{ij}{}^J \xi^{B\,J} \Pi_k{}^B B_{BCA} \\ &\quad + \tfrac{1}{3} \epsilon^{ijk} \xi^{A\,I} \Pi_j{}^C \Pi_k{}^B \Pi_i{}^D G_{DBCA} \\ &\doteq 0 \,. \end{aligned} \tag{4.14}$$

This vanishes, because the penultimate side is nothing but the multiplication of the Z^M-field equation (4.13a) by $\xi^{A\,I}$, where use has been made of the relation (4.10b).

5 Concluding remarks

In this paper, we have performed the non-Abelian gauging of the supermembrane, by introducing a vector field on its world-volume. We have confirmed that our action has three invariances, the fermionic κ-symmetry, local non-Abelian gauge symmetry, and composite Λ-symmetry for the antisymmetric tensor B_{ABC}. We have shown that the Λ-invariance requires the minimal couplings to the super Killing vector $\xi^{A\,I}$, while both Λ- and κ-invariances necessitate the Chern-Simons term, which makes our system nontrivial, but nevertheless consistent.

Since the π_3-mapping of a non-Abelian gauge group G associated with the compact space B is generally nontrivial, the m-coefficient of our Chern-Simons term is quantized. This situation is different from an Abelian gauging where $\pi_3(U(1)) = 0$. Even though the precise significance of this quantization is yet unclear, we stress that it is our formulation that reveals such a quantization in terms of supermembrane action principle in 3d.

The Abelian gauging requires a vector field on the world-volume, which is similar to the Abelian vector field used in D-branes [14]. Even though we do not yet know any direct relationships, it is quite natural to have the D-brane generalization of our formulation.

Our results in this paper bring out two important aspects of M-theory. First, the introduction of a super Killing vector $\xi^{A\,I}$ with the parameter m seems to induce *no* new physical effects on the target 11D superspace itself, because all the field strengths and BIds are entirely reduced to the original case with $m = 0$ in 11D [7]. This is also consistent with our past experience, i.e., any naïve modification of 11D supergravity [3] is bound to fail, due to the 'uniqueness' of 11D supergravity [16], unless it is related to certain M-theory higher-order correction terms. Second, most importantly, the existence of the super Killing vector $\xi^{A\,I}$ induces nontrivial physical effects on the supermembrane action in 3d, despite no seeming physical effects on the 11D target superspace. The quantization of the Chern-Simons term also supports the non-trivial feature of the system on the world-volume. To put it differently, while 11D supergravity is 'unique' [3, 7], there are still some ambiguities for supermembrane physics in the 3d world-volume. Our results have uncovered such nontrivial unknown aspects of double-compactifications of M-theory. Additionally, our formulation may well be applied to more general extended objects [15] other than supermembranes. In fact, similar Chern-Simons terms with quantizations for bosonic case for odd p-branes have been discussed in [15] based on string/5-brane duality.

To our knowledge, our formulation is the first that introduces non-Abelian minimal couplings into the supermembrane action in 11D with a Chern-Simons term. These nontrivial couplings make double-compactifications [1] more interesting, because without a supermembrane action on 3d, all the effects of t he super Killing vector $\xi^{A\,I}$ simply disappeared within the 11D target superspace. It is these non-Abelian couplings that make the new effects of $\xi^{A\,I}$ nontrivial, providing interactions with physical fields in the supermembrane action. Additionally, our non-Abelian gauge field is neither auxiliary nor composite as in past references [4], but is 'topological' with a genuine Chern-Simons term. Since the supermembrane [8] is an important 'probe' for 11D backgrounds, our result indicates important effects of super Killing vector for the compactifications on the supermembrane world-volume physics.

Since conventional Lorentz covariant superspace lacks κ-invariance in the action, and moreover, compactifications such as the one from 11D into 10D necessarily break local Lorentz symmetry, teleparallel superspace can serve as a consistent background for supermembranes with the non-Abelian super Killing vector. Supermembrane physics, as an important probe for 11D background, has revealed the necessity of teleparallel superspace in 11D or M-theory. In this sense, teleparallel superspace [13] is not just 'a technical tool', but a consistent background, when considering the double-compactification [1] of supermembranes [8].

The recent developments in 3D supergravity and supersymmetry [19] may well be closely related to the result of this paper. It is hoped that the techniques developed in this paper will play an important role, when considering general compactifications of M-theory, such as compactifications into the superstring theory in 10D or lower-dimensions.

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